Space-time localisation with quantum fields

Marc-Thierry Jaekel[a] **and Serge Reynaud**[b]

(a)Laboratoire de Physique Théorique de l'ENS[1](CNRS-UPS),
24 rue Lhomond, F75231 Paris Cedex 05
(b)Laboratoire Kastler Brossel[2](UPMC-ENS-CNRS), case 74,
4 place Jussieu, F75252 Paris Cedex 05

Abstract

We introduce observables associated with the space-time position of a quantum point defined by the intersection of two light pulses. The time observable is canonically conjugated to the energy. Conformal symmetry of massless quantum fields is used first to build the definition of these observables and then to describe their relativistic properties under frame transformations. The transformations to accelerated frames of the space-time observables depart from the laws of classical relativity. The Einstein laws for the shifts of clock rates and frequencies are recovered in the quantum description, and their formulation provides a conformal metric factor behaving as a quantum observable.

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[1]Unité propre du Centre National de la Recherche Scientifique, associée à l'Ecole Normale Supérieure et à l'Université de Paris Sud.

[2]Unité de l'Ecole Normale Supérieure et de l'Université Pierre et Marie Curie, associée au Centre National de la Recherche Scientifique.

The formulation of relativistic theories relies on the concept of events localised in space-time. As a seminal example, the relativistic property of time was introduced by Einstein when he questionned the notion of simultaneity and defined synchronisation through the transfer of light pulses between remote clocks [1]. A key point in this operational approach is the possibility of associating space-time positions with events such that emission or reception of a pulse. Such positions necessarily are physical observables which have to be distinguished from coordinate parameters on a map of space-time. In particular, the relativistic properties of these observables result from a symmetry of the laws of physics, namely their Lorentz invariance. A naive identification of these properties with map transformations could only be an oversimplifying and misleading shortcut. This is the reason why the interpretation of relativity, as embodied in symmetry principles, has been repeatedly upheld against the more common and substantially different interpretation of relativity, as rooted in covariance principles [2].

While the previous arguments refer to classical theories of relativity, the concept of space-time localisation is known to raise challenging issues in the context of quantum theory [3]. In standard quantum formalism, time is never treated as an operator, and thus has a quite different description from that of space position. Moreover, the very notion of quantum fluctuations of time remains a matter of debate, since the formalism does not provide a precisely stated energy-time commutation relation which would assert that the fourth Heisenberg inequality effectively constrains time and energy fluctuations [4]. Differences in the description of space and time variables and the subsequent inconsistency between the formalism of quantum theory and the geometric description of space-time are also known to be knotty points in attempts to include gravity in quantum theory [5].

The basic idea underlying the present work is that space-time observables certainly belong to the quantum domain, like clocks used for time definition and electromagnetic signals used for synchronisation. Metrological considerations support this idea, since the definition of space-time units is now rooted in atomic physics [6]. These arguments imply that relativity theory has to be consistent with a quantum description of space-time observables. They also point to the need for a clear distinction between two commonly confused notions of time. On one hand, time is the basic evolution parameter used to write dynamical laws and conservation laws. On the other hand, time is the physical variable delivered by a clock when a given physical event occurs. The former notion of time is clearly distinct from any spatial variable while the latter one is an extension of the concept of localisation in space to that of localisation in time. It is this latter definition of time, and not the former one, which is mixed with that of space by relativistic transformations. As already noticed, these relativistic transformations have to be considered as laws of physics and not simply as direct consequences of map transformations. In particular, the shifts of space-time observables under transformations to ac-

celerated frames [7] cannot be deduced from covariance properties associated with the corresponding map transformations.

In this respect, the case of uniform acceleration requires a specific attention, since there exist conformal coordinate transformations which fit accelerated motion and still preserve propagation equations of electromagnetic field [8]. This property is a particular case of conformal invariance of massless field theories [9]. The propagation of such fields is not sensitive to a conformal variation of the metric tensor, that is a change of space-time scale preserving the velocity of light [10]. Since this conformal symmetry enlarges the symmetry built on Lorentz invariance and inertial motions, it may be expected that it allows to determine the relativistic properties of space-time observables for uniformly accelerated observers, in the same manner as Lorentz invariance for inertial observers. Indeed, preliminary results in this direction have been obtained for the problem of clock synchronisation performed through the transfer of a light pulse. The time reference encoded in the pulse, which has to be shared by two remote observers, must be a quantity preserved by field propagation. Such a quantity may actually be built from conformal generators associated with the field state and its relativistic properties may then be deduced from the conformal algebra [11].

Following this constructive approach, we show in the present letter that quantum space-time observables may be associated with a physical event defined by the intersection of two light pulses and that these definitions fulfill correct quantum and relativistic properties. First, these observables will be found to obey canonical conjugation relations with momentum operators. In particular, time will be defined as a quantum observable with fluctuations obeying an energy-time Heisenberg relation. Moreover, the definitions given in this letter will describe time and space observables in an explicitly Lorentz invariant manner. Then, the relativistic properties of space-time and energy-momentum observables as seen by inertial or accelerated observers will be derived from the conformal algebra. These properties, which will generalise covariance rules to the quantum domain, will be seen to depart from the statements inherited from classical relativity.

Before coming to a fully quantum mechanical discussion, we will introduce the necessary distinction between space-time observables and coordinate parameters in a classical context. This will allow us to show how space-time observables are constructed from the generators associated with conformal symmetry and how their relativistic properties under frame transformations are obtained from the same symmetry. We first define Lie transformations, which represent changes of frame as deformations of the coordinate map

$$x^\mu \xrightarrow{a} \overline{x}^\mu = x^\mu + \varepsilon_a \delta_a^\mu(x) \tag{1}$$

where ε_a is an infinitesimal number and δ_a^μ a polynomial function of coordinate parameters x. The commutator between two frame transformations is thus represented by the difference between the images of a point x through

the composed deformations $(a \circ b)$ and $(b \circ a)$, which is evaluated as $\varepsilon_a \varepsilon_b \delta^\mu_{(a,b)}(x)$ where $\delta^\mu_{(a,b)}$ is the Lie commutator

$$\delta^\mu_{(a,b)} = \delta^\nu_b \partial_\nu \delta^\mu_a - \delta^\nu_a \partial_\nu \delta^\mu_b \tag{2}$$

We shall assume that frame transformations are performed around an inertial frame and raise or lower tensor indices by using the Minkowski tensor $\eta_{\mu\nu}$. However, these transformations give in general rise to a change of the metric tensor

$$g_{\mu\nu} = \eta_{\mu\nu} \xrightarrow{a} \overline{g}_{\mu\nu} = \eta_{\mu\nu} - \varepsilon_a \left(\partial_\mu \delta_{a\nu} + \partial_\nu \delta_{a\mu} \right) \tag{3}$$

For Lorentz transformations, the metric tensor is unchanged. For conformal transformations, its change reduces to a point-dependent rescaling

$$\begin{aligned} g_{\mu\nu} = \eta_{\mu\nu} \xrightarrow{a} \overline{g}_{\mu\nu} &= \eta_{\mu\nu} \left(1 + 2\varepsilon_a \lambda_a(x) \right) \\ 2\lambda_a \eta_{\mu\nu} &= -\partial_\mu \delta_{a\nu} - \partial_\nu \delta_{a\mu} \end{aligned} \tag{4}$$

Conformal transformations are thus associated with the preservation of the velocity of light and, consequently, of the propagation of massless fields [8]. The commutator (2) of conformal transformations is still a conformal transformation, and the set of these commutators constitutes the conformal algebra which characterises the symmetry properties of the field theory [15]. The associated conserved quantities, that is the generators of the symmetries, are used in the following to characterise the field states while the conformal algebra determines the transformations of these quantities and therefore the transformations of the field states. The prime role is thus played by the conformal generators which correspond to translations (P_ν), rotations $(J_{\nu\rho})$, dilatation (D) and conformal transformations to uniformly accelerated frames (C_ν) and are given respectively by the following deformations [9]

$$\begin{aligned} \delta^\mu_{P_\nu}(x) &= \eta^\mu_\nu & \lambda_{P_\nu}(x) &= 0 \\ \delta^\mu_{J_{\nu\rho}}(x) &= \eta^\mu_\nu x_\rho - \eta^\mu_\rho x_\nu & \lambda_{J_{\nu\rho}}(x) &= 0 \\ \delta^\mu_D(x) &= x^\mu & \lambda_D(x) &= -1 \\ \delta^\mu_{C_\nu}(x) &= 2x_\nu x^\mu - \eta^\mu_\nu x_\rho x^\rho & \lambda_{C_\nu}(x) &= -2x_\nu \end{aligned} \tag{5}$$

η^μ_ν denotes a Kronecker symbol.

We then consider a classical light ray defined as a dispersionless field distribution running along a light ray parametrised as

$$x^\mu = \xi^\mu + p^\mu \sigma \qquad p^\mu p_\mu = 0 \tag{6}$$

where ξ^μ represents any origin on the ray, p^μ a light-like momentum vector and σ an affine parameter along the ray. This parametrisation relies on the assumption of a dispersionless distribution both in momentum and position spaces. This assumption, clearly inconsistent with the principles of quantum

physics, will be released in the following. In this restricted context however, it allows us to write in a simple manner the conformal generators

$$\Delta = p_\nu \delta^\nu(x) \tag{7}$$

The conservation laws for these generators then appear as a direct consequence of definitions (4) and (6)

$$\frac{\mathrm{d}\Delta}{\mathrm{d}\sigma} = p_\nu p^\mu \ \partial_\mu \delta^\nu = -\lambda \ p_\mu p^\mu = 0 \tag{8}$$

We can now make clear that the evolution parameter σ is distinct from any conceivable notion of time observable. As a matter of fact, the values of the generators Δ are conserved, i.e. independent of σ, on each light ray whereas they clearly vary with variables describing the position of the ray in space-time. In particular, they are changed when the origin ξ_μ of the light ray is displaced

$$\mathrm{d}\Delta = p_\nu \ \mathrm{d}\xi^\mu \ \partial_\mu \delta^\nu(\xi) \neq 0 \tag{9}$$

except when the displacement $\mathrm{d}\xi^\mu$ is parallel to the momentum and thus preserves the ray. The last equation gives the change $\mathrm{d}\Delta$ of conformal generators when the light ray is translated in a given reference frame, but also when the frame is translated, provided that the signs are carefully taken care of.

We now describe the ray transformations in a more general manner. We first characterize the various rays by the values of the generators (7). This characterisation is intrinsic since it does not rely upon an arbitrary parametrisation such as the one of equation (6). Conversely, such a parametrisation may be derived, if necessary, from the values of the momentum and rotation generators. The set of classical light rays is then mapped into itself by the conformal transformations, and this mapping is described by the conformal algebra. If one denotes Δ_a the generator associated with a transformation and Δ_b a conserved quantity associated with a ray, the change of Δ_b under the transformation Δ_a may be read as $\varepsilon_a \Delta_{(a,b)}$ where $\Delta_{(a,b)}$ is the conformal generator given by the commutator (2)

$$\Delta_{(a,b)} = p_\nu \delta^\nu_{(a,b)} = \delta^\mu_b \partial_\mu \Delta_a - \delta^\mu_a \partial_\mu \Delta_b \tag{10}$$

The particular case (9) of changes $\mathrm{d}\Delta$ under translations is now embodied in the commutator between P_μ and Δ. Since a commutator is antisymmetric in the exchange of its two arguments, this change is also the opposite of the change of momentum P_μ under the action of Δ.

The discussion restricted up to now to the case of a classical light ray shows that the conformal symmetry allows to characterise the rays as well as to describe the effects of frame transformations. We may stress that a field state containing a single light ray is associated with a geometrical line rather than with a point. In contrast, a field state comprising two rays can

be used to obtain the position of a localised event defined as the point of intersection of the two lines. We therefore consider now a state built with two field pulses coinciding at a space-time position X^μ and propagating in different directions defined by momenta $p_\pm^\mu$. Each of the two rays may be parametrised by equation (6) with both origins chosen at the coincidence point X^μ. Conserved quantities (7), defined as sums of the contributions of the two rays, thus read

$$\Delta = P_\nu \delta^\nu(X) \tag{11}$$

where P_ν is the total energy-momentum of the field state $\sum_\pm p_{\nu\pm}$.

Since the elementary rays have different propagation directions, the mass associated with the field state

$$M^2 = P_\nu P^\nu \tag{12}$$

no longer vanishes. The expressions of the rotation and dilatation generators $J^{\mu\nu}$ and D may therefore be inverted to obtain the space-time position of the coincidence point

$$\begin{aligned} J^{\mu\nu} &= P^\mu X^\nu - P^\nu X^\mu \qquad D = P_\mu X^\mu \\ X^\mu &= \frac{P^\mu}{M^2} D - \frac{P_\nu}{M^2} J^{\mu\nu} \end{aligned} \tag{13}$$

We may emphasize that the connection between the definition of an observable space-time position X^μ and the massive character of the state has a simple physical interpretation, in the spirit of the discussion about localisation already presented. A vanishing mass M corresponds to a field state with a single propagation direction which cannot be associated with a localised event. In contrast, a non vanishing mass reveals that the state contains different light rays which intersect and thus define a point in space-time. It is also worth stressing that the observable position X is built from conserved quantities and, consequently, does not evolve due to field propagation

$$\frac{\mathrm{d}X^\mu}{\mathrm{d}\sigma} = 0 \tag{14}$$

This confirms that space-time observables conceptually differ from coordinate parameters, for example from those associated with field pulses running along trajectories. In particular, the temporal component X^0 must not be confused with the affine parameter σ. As already discussed, a variation of σ describes pulse propagation along the trajectories while a variation of X^0 corresponds to a translation of the trajectories and therefore of the time observable associated with the coincidence event. More generally, changes of frame are described by the conformal algebra (10). Notice that the commutators between the conformal generators may also be written as Poisson

brackets which generalise the Lie commutators initially defined for map deformations to observables [12]

$$\Delta_{(a,b)} \equiv (\Delta_a, \Delta_b) = \frac{\partial \Delta_a}{\partial X^\mu} \frac{\partial \Delta_b}{\partial P_\mu} - \frac{\partial \Delta_a}{\partial P_\mu} \frac{\partial \Delta_b}{\partial X^\mu} \tag{15}$$

We come now to the quantum mechanical discussion where the distinction between observables and evolution parameter will become even more striking than in the classical context. Space-time observables will indeed appear as quantum operators, whereas σ will remain a classical evolution parameter. In the quantum discussion, the field states exhibit momentum and position dispersions as a consequence of Heisenberg relations. Conformal generators are defined as integrals of operators $T_{\mu\nu}$ representing components of the stress tensor

$$\Delta = \int_\sigma \mathrm{d}\Sigma \; T_{\mu 0}(x) \; \delta^\mu(x) \tag{16}$$

The symbol $\int_\sigma \mathrm{d}\Sigma$ denotes an integral over a space-like surface at constant coordinate parameter σ. The energy-momentum densities $T_{\mu 0}$ are normally ordered so that they vanish in vacuum. Notice that the conformal transformations (5) do not only preserve the propagation equation, but also the definition of vacuum [13] and of particle number [14]. Noether's theorem asserts [15] that the generators (16) are preserved by field propagation like the classical ones (see (8)). The canonical field commutators and the definition of stress tensor are such that the quantum commutators of the conformal generators (16) identify with the Lie commutators [12]

$$\frac{1}{i\hbar} [\Delta_a, \Delta_b] = (\Delta_a, \Delta_b) \tag{17}$$

For this reason, we will further use the notation $(\ ,\ )$ rather than $\frac{1}{i\hbar}[\ ,\ ]$ for writing the commutation relations.

Now, observable positions in space-time may be defined from the conformal generators. As a matter of fact, centers of inertia of the energy-momentum distribution may be obtained by inverting expressions of rotation and dilatation generators $J^{\mu\nu}$ and D as in classical equations (13)

$$\begin{aligned} J^{\mu\nu} &= P^\mu \cdot X^\nu - P^\nu \cdot X^\mu \qquad D = P_\mu \cdot X^\mu \\ X^\mu &= \frac{P^\mu}{M^2} \cdot D - \frac{P_\nu}{M^2} \cdot J^{\mu\nu} \end{aligned} \tag{18}$$

We have taken care of non commutativity of quantum observables by introducing a symmetrised product represented by the " $\cdot$ " symbol. We have also assumed that the mass (12) associated with the field state does not vanish. The definition (18) of space-time positions associated with the field state is quite analogous to Einstein's classical definition of spatial positions [16]. However, it involves not only the rotation generators $J^{\mu\nu}$ but also the dilatation generator D. As a result, an observable time is defined together

with space positions. Furthermore, the definition (18) holds in the quantum domain, with the particularly important outcome that the space-time observables are canonically conjugated to energy-momentum operators

$$(P^\mu, X^\nu) = -\eta^{\mu\nu} \tag{19}$$

Hence, the commutators of functions of energy-momentum and space-time observables may equivalently be written as Poisson brackets (15). The space-time observables are not themselves conformal generators and thus do not belong to the conformal algebra. However, the canonical commutation relations (19) involve the enveloping algebra, that is the structure built on polynomial functions of the conformal generators. In this sense, canonical commutation relations can be considered as embodied in conformal symmetry. It is worth emphasizing that an energy-time Heisenberg relation is now obtained besides momentum-space relations of standard quantum formalism. Furthermore, these relations enter a Lorentz invariant description.

Relations (18) imply that the generators $J_{\mu\nu}$ and D have a classical form in terms of the observable X defined as center of the energy-momentum distribution. This is related to the fact that the corresponding deformations δ^μ are linear functions of x. Since transformations to accelerated frames correspond to quadratic functions, the generators C_ν will differ from the corresponding classical form because of dispersions associated with Heisenberg relations [11]. It is thus natural to write the various generators as

$$\begin{aligned} \Delta &= P_\mu \cdot \delta^\mu(X) + \widehat{\Delta} \\ \widehat{P}_\nu &= \widehat{J}_{\nu\rho} = \widehat{D} = 0 \qquad \widehat{C}_\nu \neq 0 \end{aligned} \tag{20}$$

The first contribution to Δ has a classical form to be compared with (11). The second contribution $\widehat{\Delta}$ is thus defined as a correction to the classical expression which differs from 0 only for transformations to accelerated frames. We may then check by inspection of the conformal algebra (10) that the commutators of any generator Δ with a translation generator P_μ belong to the set of generators which have a classical form

$$(\Delta, P_\mu) = P_\nu \cdot \partial_\mu \delta^\nu(X) \tag{21}$$

This proves that the corrections $\widehat{\Delta}$ always commute with the momentum operators

$$\left(\widehat{\Delta}, P_\mu\right) = \frac{\partial \widehat{\Delta}}{\partial X^\mu} = 0 \tag{22}$$

Therefore the non vanishing corrections $\widehat{C}_\nu$ may be written in terms of the momentum operators and of the Casimir invariants of the conformal algebra. Dimensional analysis implies that $\widehat{C}_\nu$ scales as the inverse of a momentum and is thus a non-linear expression of momentum. To specify the argument,

we will assume that the field state consists of two intersecting light rays with identical dispersions. In this case, the correction $\widehat{C}_\nu$ has the simple form

$$\widehat{C}_\nu = \alpha \frac{P_\nu}{M^2} \tag{23}$$

where α is a Casimir invariant of the conformal algebra. It is related to the Casimir invariants associated with each light ray [11] and is positive with a minimal magnitude of the order of $\hbar^2$.

Having defined space-time observables in terms of the conformal generators, we come now to the next stage of the analysis, devoted to their transformations under changes of frame. The commutator (Δ, P_μ) characterises the momentum change under the frame transformation associated with Δ, as it results from a comparison with classical relations (9,15). Equations (21) thus mean that this momentum change has precisely the form expected for the shift of a vector field in classical differential geometry. This is true not only for Lorentz transformations, but also for dilatations and transformations to accelerated frames. In particular, the Einstein prediction of a position dependent momentum change for transformations to accelerated frames [7] is recovered in the context of quantum theory. Moreover, the dependence of this change in terms of observable positions is the same as for the classical expression written in terms of coordinate parameters.

This perfect matching between quantum and classical laws still holds for the shift of space-time observables X^μ under translations, rotations and dilatations, but not under transformations to accelerated frames. Using equation (20), we indeed deduce the following quantum law for the shifts of space-time observables

$$(\Delta, X^\mu) = -\delta^\mu(X) - \frac{\partial \widehat{\Delta}}{\partial P_\mu} \tag{24}$$

The first classically-looking term is corrected by the second term which differs from 0 only for transformations to accelerated frames. In the particular case of two identical intersecting rays, we obtain this correction from (23)

$$-\frac{\partial \widehat{C}_\nu}{\partial P_\mu} = -\frac{\alpha}{M^2}\left(\eta^\mu_\nu - \frac{2P^\mu P_\nu}{M^2}\right) \tag{25}$$

Hence, the shifts (24) of space-time observables do not obey the covariance rules inherited from classical relativity, and the corrections to these rules follow from conformal algebra. The classical approach presented in the first part of this letter was an approximation where light rays were treated as dispersionless distributions. This approximation amounts to set the Casimir operators to 0, in which case the correction vanishes so that observables are transformed as classical parameters. For quantum fields in contrast, uncertainty relations forbid to set the Casimir operators to 0 and the transformations of observables effectively differ from those of coordinate parameters [11].

In particular, space-time observables are mixed with momentum operators under frame transformations.

For transformations to uniformly accelerated frames, we have found that the shifts of space-time observables do not have a classically looking form, in contrast to those of energy-momentum operators. It is however possible to write down consistency statements for these shifts which allow to make contact with classical laws. Such relations follow from the fact that the canonical commutators (19) are invariant under frame transformations, since they are classical numbers

$$(\Delta, (P^\mu, X^\nu)) = 0 \tag{26}$$

It then follows from Jacobi identity that variations of space-time and energy-momentum shifts are connected through

$$(P^\mu, (\Delta, X^\nu)) = (X^\nu, (\Delta, P^\mu)) \tag{27}$$

Both types of variations therefore have a classically-looking form (see (21) and (24))

$$-\frac{\partial}{\partial X_\mu}(\Delta, X^\nu) = \frac{\partial}{\partial P_\nu}(\Delta, P^\mu) = \partial^\mu \delta^\nu(X) \tag{28}$$

In the particular case $\mu = \nu = 0$, the first quantity is the shift of a clock rate, while the second quantity is the change of the redshift of frequency. Relations (28) express both quantities in terms of the function $\partial^0\delta^0$ that is also the metric coefficient g^{00} in equation (4). These results prove that the relativistic transformations of space-time scales and energy-momentum shifts are now consistently obtained in the quantum domain from the same commutation relations. The close connection between relations (28) and the metric tensor has in fact more general implications. Quantum analogs of the definition (4) of the conformal factor are indeed obtained by symmetrising expression (28) in the exchange of the two indices μ and ν

$$\begin{aligned} \frac{\partial}{\partial X_\mu}(\Delta, X^\nu) + \frac{\partial}{\partial X_\nu}(\Delta, X^\mu) &= 2\eta^{\mu\nu}\lambda(X) \\ \frac{\partial}{\partial P_\mu}(\Delta, P^\nu) + \frac{\partial}{\partial P_\nu}(\Delta, P^\mu) &= -2\eta^{\mu\nu}\lambda(X) \end{aligned} \tag{29}$$

The classical definition (4) was written in terms of classical coordinate parameters and it was therefore not properly defined from an operational point of view. In contrast, the quantum definition (29) is expressed in terms of observables and may thus be deduced from measurements of field quantities. It is a remarkable consequence of the consistency statement discussed in the previous paragraph that the conformal factor may be deduced from measurements of space-time or energy-momentum observables. It is also remarkable

that the conformal factor appears in the transformation laws (29) of space-time observables, although propagation of electromagnetic field is known to be insensitive to a conformal variation of the metric tensor [10].

In the present letter we have introduced quantum observables describing the space-time position of the physical event defined by the intersection of two light pulses and we have shown that these observables are canonically conjugated to energy-momentum operators associated with the same field state. In particular, a time observable has been defined which is canonically conjugated to energy. The shifts of these observables under transformations to accelerated frames have been derived from conformal algebra and shown to depart from the laws of classical relativity, since space-time and energy-momentum are mixed. The Einstein laws for transformations of clock rates and frequency redshift have nevertheless been recovered and expressed in terms of the conformal factor.

In more general words, these results introduce a new conception of space built on the notion of quantum points. These points are the analogs in quantum theory of the localised events of relativity theory. The relativistic properties of this space are determined by the conformal symmetry which underlies quantum theory of massless fields. The metric coefficients which may be deduced from measurements of space-time or energy-momentum observables must also be considered as quantum operators. These results clearly point to the need for a quantum geometry [17].

References

[1] A. Einstein, Annalen der Physik **17** (1905) 891.

[2] J.D. Norton, Reports on Progress in Physics **56** (1993) 791.

[3] Examples of discussions of this point may be found in T.D. Newton and E. Wigner, Review of Modern Physics **21** (1949) 400; H. Salecker and E.P. Wigner, Physical Review **109** (1958) 571.

[4] A detailed historical account of discussions on this problem may be found in M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley, NewYork, 1974) ch.5.

[5] C. Rovelli, Physical Review **D42** (1990) 2638, **D43** (1991) 442; Classical and Quantum Gravity **8** (1991) 297, 317.

[6] See for example the Special Issue on Time and Frequency in Proceedings of IEEE **79** (1991) 891-1079.

[7] A. Einstein, Jahrb. Radioakt. Elektron. **4** (1907) 411.

[8] H. Bateman, Proceedings of the London Mathematical Society **8** (1909) 223; E. Cunningham, ibidem **8** (1909) 77.

[9] T. Fulton, F. Rohrlich and L. Witten, Nuovo Cimento **26** (1962) 653; Review of Modern Physics **34** (1962) 442 and references therein.

[10] B. Mashhoon and L.P. Grishchuk, Astrophysical Journal **236** (1980) 990.

[11] M.T. Jaekel and S. Reynaud, Physical Review Letters **76** (1996) 2407.

[12] B. de Witt, Review of Modern Physics **29** (1957) 377.

[13] M.T. Jaekel and S. Reynaud, Quantum and Semiclassical Optics **7** (1995) 499.

[14] M.T. Jaekel and S. Reynaud, Brazilian Journal of Physics **25** (1995) 315.

[15] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw Hill, New York, 1985).

[16] A. Einstein, Annalen der Physik **20** (1906) 627.

[17] A. Connes in *Gravitation and quantizations*, eds B. Julia and J. Zinn-Justin (1995) 805.